

What is a logarithm?

- To answer this, first try to answer the following:
what is x in this equation? $9 = 3^x$
what is x in this equation? $8 = 2^x$
- Basically, logarithmic transformations ask, “a number, to **what power** equals another number?”
- In particular, logs do that for specific numbers under the exponent.

Natural logs, or log base e.

- Why e?

$$\begin{aligned} &= 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \quad (\text{remember: } 3! = (3)(2)(1)) \\ &= \text{the limit of } (1 + \frac{1}{n})^n \text{ as } n \rightarrow \infty \\ &= 2.718281828459045235\dots \end{aligned}$$

In 1864 Benjamin Peirce would write $e^x = \sqrt[\#]{\quad}$ and say to his students:

“We have not the slightest idea what this equation means, but we may be sure that it means something very important.”

- e has the simplest derivative:

Examples:

$$(e^{45}) = 45$$

$$\log(10^{23} \times 10^{45}) = 68$$

$$10^{23} (e^{46}) = 46 \times 10^{23}$$

- Solve the following for x:
 $\log(256/x) = 1.5$
 $(256/x) = 10^{1.5}$

Graphing with logarithms

Another powerful use of logarithms comes in graphing. For example, exponential functions are tricky to compare visually. It's hard to see what happens at small values and at large values at the same time because the function increases (or decreases) so quickly. To help with this, we sometimes plot the log of a function. For example, look at the two functions in this graph:

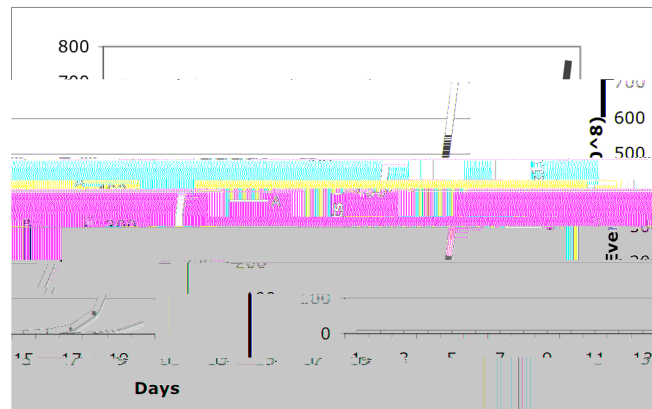


Figure 2. A very unhelpful plot of the frequency of some events over time.

For our purposes it doesn't much matter what the two functions are, but we can see that if we graph both A and B on the same plot, we see that we have almost no idea what's happening below ~15 days on the x-axis and we almost can't see the plot of A because the scale of B is so much greater.

Now, take the same two functions, but this time plot the log (base 10 in this case) of each function:

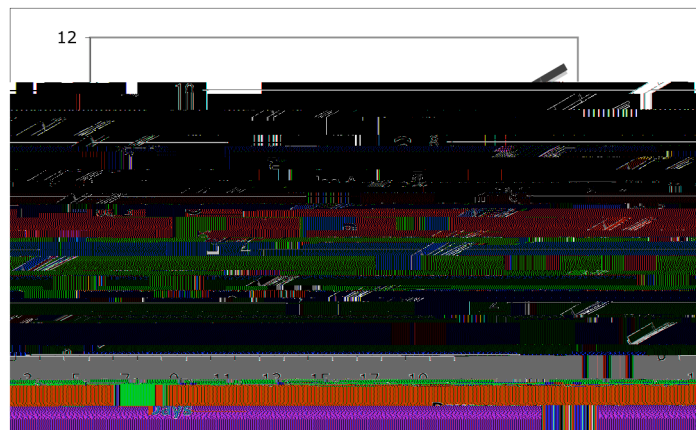


Figure 3. The same data from Figure 2, presented as a log plot.

Already it is easier to compare the two and we gain more insight as to the properties of the function at both high and low ranges. Notice also that the function has become linear.

The use of logarithms in graphing can also show us important details in exponential functions that may remain hidden otherwise. For example, look at the following data:

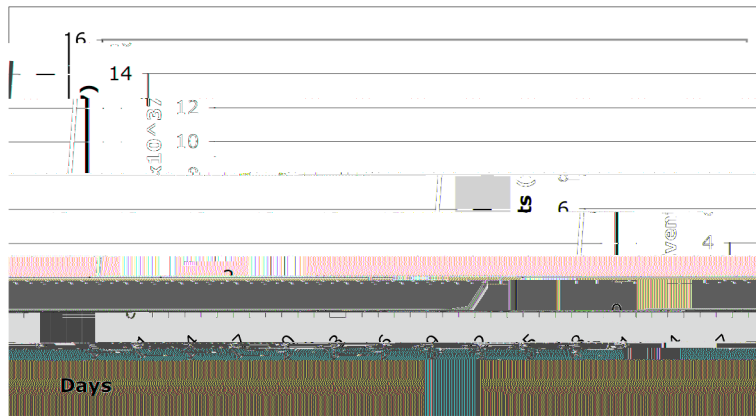


Figure 4. Another unhelpful plot of an exponential function.

While this plot is not so informative, see what appears if we plot the logarithm (again, base 10 in this case):

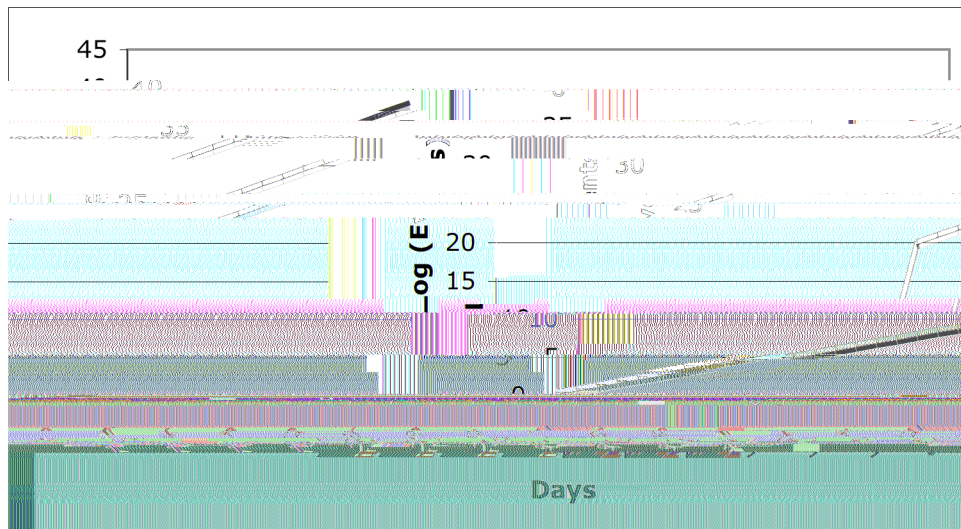


Figure 5. The same data from Figure 4 presented as a log plot.

Now we can see that there are TWO distinct processes occurring here and that there is